Week 1 HW Quiz: Tuesday, 4/3

p. 570, MC practice Part B, FR # 4 - 6

Calculus Warm Up #2-2

1) Verify the ∞ Series diverges:

$$\sum_{n=1}^{\infty} \frac{n}{2n+3}$$

2) Verify the ∞ Series converges, then find the sum

$$\sum_{n=0}^{\infty} 2\left(-\frac{3}{4}\right)^{n}$$

HW Questions: p. 579

1. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$

3. $3 - \frac{9}{2} + \frac{27}{4} - \frac{81}{8} + \frac{243}{16} - \cdots$

5. $\sum_{n=1}^{\infty} \frac{3}{2^{n-1}}$

verify diverges:

7. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots$

9. $\sum_{n=1}^{\infty} \frac{3n}{n+1} = \frac{3}{2} + \frac{6}{3} + \frac{9}{4} + \frac{12}{5} + \cdots$

11. $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$

13. $\sum_{n=0}^{\infty} 3\left(\frac{3}{2}\right)^n$

15. $\sum_{n=0}^{\infty} 1000(1.055)^n$

17. $\sum_{n=1}^{\infty} \frac{2^n + 1}{2^{n+1}}$ $\Rightarrow \lim_{n\to\infty}\left(\frac{\cancel{2^n}}{\cancel{2^n}\cdot 2^1}\right) + \lim_{n\to\infty}\left(\frac{1}{2^n \cdot 2^1}\right)$

$= \frac{1}{2} + \frac{1}{\cancel{2^\infty}} \nearrow 0$

$= \frac{1}{2}$

Series diverges b/c $\lim_{n\to\infty} a_n = \frac{1}{2} \neq 0$

verify converges:

19. $2 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128} + \cdots$

21. $\sum_{n=0}^{\infty} (0.9)^n = 1 + 0.9 + 0.81 + 0.729 + \cdots$

23. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (Use partial fractions.)

25. $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$

27. $\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n$

29. $1 + 0.1 + 0.01 + 0.001 + \cdots$

31. $3 - 1 + \frac{1}{3} - \frac{1}{9} + \cdots \rightarrow r = -\frac{1}{3}$

32. $4 - 2 + 1 - \frac{1}{2} + \cdots$

33. $\sum_{n=4}^{\infty} 3\left(\frac{5}{8}\right)^n \longrightarrow r = \frac{5}{8}$

$|r| = \frac{1}{3} < 1$

converges.

$S = \frac{3}{1-\left(-\frac{1}{3}\right)} = \frac{9}{4}$

$a_1 = 3\left(\frac{5}{8}\right)^4$

35. $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ →

37. $\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$

39. $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right)$

$1 = A(n-1) + B(n+1)$

$A = -\frac{1}{2}$, $B = \frac{1}{2}$

$$\sum_{n=2}^{\infty}\left(-\frac{1}{2(n+1)} + \frac{1}{2(n-1)}\right)$$

$n=2$, $n=3$, $n=4$, $n=5$

$$= \frac{1}{2}\left(\left(-\frac{1}{3}+1\right) + \left(-\frac{1}{4}+\frac{1}{2}\right) + \left(-\frac{1}{5}+\frac{1}{3}\right) + \left(-\frac{1}{6}+\frac{1}{4}\right)\right.$$

$$= \frac{1}{2}\left(1+\frac{1}{2}\right)$$

$$= \frac{3}{4}$$

39 $\sum\left(\frac{1}{2}\right)^n + \sum\left(\frac{1}{3}\right)^n$

41. $0.6\overline{66}$

43. $0.07\overline{575}$ $= 0.075 + 0.00075 + 0.0000075 \dots$

$r = 10^{-2} = 0.01$

$$S = \frac{0.075}{1-0.01} = \frac{0.075}{0.99} = \frac{5}{66}$$

let $x = 0.0\overline{75}$

$100x = 7.5\overline{75}$

$-\ \ x \quad -0.0\overline{75}$

$\frac{99x}{99} = \frac{7.5}{99}$

$x = \frac{5}{66}$

Partial Sums

The partial sums of a series form a sequence of real #'s: $S_1, S_2, S_3, \ldots$

If the sequence of partial sums has a limit, then the original series converges.

The nth Term Test for Divergence

Whenever an infinite series converges, the limit as $n \to \infty$ of the nth term = 0

If $\lim_{n \to \infty} a_n \neq 0$, then the series diverges.

Classwork:

Yellow FR Practice

HW: p. 579, # 45 - 55 odd

and work on yellow classwork